1 Introduction

[1]

2 Wynn's ϵ-algorithm

Wynn [2] developed the ϵ-algorithm based on Shanks' series. It goes as follows. Given a sequence of partial sums $\{s_n\}$ with $n = 1, 2, \ldots, N$, define

$$\epsilon_{-1}(s_n) = 0\,, \qquad\qquad \epsilon_0(s_n) = s_n\,. \tag{1}$$

Then,

$$\epsilon_{j+1}(s_n) = \epsilon_{j-1}(s_{n+1}) + \frac{1}{\epsilon_j(s_{n+1}) - \epsilon_j(s_n)} \tag{2}$$

for $j = 0, 1, 2, \ldots$. The $\epsilon_{2j}(s_n)$ are approximations to the series.

The ϵ-table is of the form

$$\begin{array}{ccccccc}
0 & s_1 & \epsilon_1(s_1) & \epsilon_2(s_1) & \epsilon_3(s_1) & \epsilon_4(s_1) & \ldots \\
0 & s_2 & \epsilon_1(s_2) & \epsilon_2(s_2) & \epsilon_3(s_2) & \ldots & \ldots \\
0 & s_3 & \epsilon_1(s_3) & \epsilon_2(s_3) & \ldots & \ldots & \ldots \\
0 & s_4 & \epsilon_1(s_4) & \ldots & \ldots & \ldots & \ldots \\
0 & s_5 & \ldots & \ldots & \ldots & \ldots & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array} \tag{3}$$

The best approximations are usually taken to be $\epsilon_{2j}(s_1)$ for odd N, or $\epsilon_{2j}(s_2)$ for even N.

To implement, we need two arrays for even and add j. To generte the next j, we can overwrite the very first entry in the older array.

References

[1] Henri Cohen, Fernando Rodriguez Villegas, and Don Zagier. Convergence acceleration of alternating series. *Experimental Mathematics*, 9(1):3–12, 2000.

[2] P. Wynn. On a device for computing the em(sn) transformation. *Mathematical Tables and Other Aids to Computation*, 10(54):91–96, 1956.